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Leading nucleons in nucleon–air collisions

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Abstract. We present in this paper a calculation of the hadronic flux in the atmosphere. Using an iterative leading-particle model in the Glauber framework, we relate the moment of the leading-particle distribution in nucleon–air collisions with its counterpart one in nucleon–proton collisions.

1 Introduction

Analytical solutions for the nucleonic diffusion equation in the atmosphere, having as a boundary condition the primary spectrum and calculated with the leading-particle model, show a strong correlation between the inelastic proton–air cross section and the momentum of the leading-particle distribution. For the experimental data to be analyzed, the behavior, along with energy of the inelastic cross section and the leading-particle distribution, must be known.

Concerning the inelastic cross section, at low energy, it can be obtained from experimental data of proton–nucleus scattering [1]. With the Glauber model assumed to describe multiple proton–air collisions, the inelastic proton–air cross section can be also derived through the use of accelerator experimental data on total proton–proton cross section [2]. Beyond the accelerator energy region, parametrizations for the total pp cross section can be used to extrapolate to the high energy region.

The situation concerning the leading-particle distribution is more complicated. The leading-particle distribution for proton–air collisions cannot be derived in a simple way from cosmic ray experimental data. Some phenomenological models have been applied for the leading nucleon distribution to explain experimental data on the nucleonic flux in the atmosphere [3–5], in analogy with the proton–proton scattering.

The leading-particle spectrum for hadron–nucleus collisions was studied only at low energy. The proton leading-particle spectrum was studied at only the ISR (Intersecting Storage Rings) [6] energy region, and showed a flat distribution and mean inelasticity near 0.5. But, unfortunately, there are no similar analyses coming from Cern and Tevatron Colliders.

For proton–nucleus scattering, at low energy, several models for describing the leading-particle spectrum have been proposed (the interacting gluon model and Regge–Mueller formalism) [7–9]. Here, we shall work in the iterative leading-particle model [10,11] and use the notation of Frichter, Gaisser, and Stanev [12]. In this model, the leading-particle spectrum in $\mathrm{p} + \mathrm{A} \to \mathrm{N}(\text{nucleon}) + X$ collisions is built from successive interactions with ν interacting protons of the nucleus A, and the behavior is controlled by a straightforward convolution equation. It should be mentioned that, strictly speaking, the convolution should be 3-dimensional. Here we consider only the 1-dimensional approximation.

2 Nucleon–air collisions

Considering the longitudinal distribution: For multiple scattering of incident nucleons with nucleons inside the nucleus, after ν collisions, the longitudinal distribution is obtained by means of the Mellin convolution integral

$$M_\nu^{\mathrm{p}}(x) = \int_x^1 \frac{\mathrm{d}y}{y}[S_{\nu-1}^{+}(y)\beta_{\nu-1}M_{\nu-1}^{\mathrm{p}}(x/y) + S_{\nu-1}^{-}(y)(1-\beta_{\nu-1})M_{\nu-1}^{\mathrm{n}}(x/y)] \quad (1)$$

for protons, and

$$M_\nu^{\mathrm{n}}(x) = \int_x^1 \frac{\mathrm{d}y}{y}[S_{\nu-1}^{+}(y)\beta_{\nu-1}M_{\nu-1}^{\mathrm{n}}(x/y) + S_{\nu-1}^{-}(y)(1-\beta_{\nu-1})M_{\nu-1}^{\mathrm{p}}(x/y)] \quad (2)$$

for neutrons. $M_\nu^{\mathrm{p,n}}(x)$ are the proton and neutron distributions, normalized as

$$\int_0^1 \mathrm{d}x M_\nu^{\mathrm{p,n}}(x) = n_\nu^{\mathrm{p,n}} \quad (3)$$

with $n_\nu^{\mathrm{p}} + n_\nu^{\mathrm{n}} = 1$. The numbers n_ν^{N} express the outgoing nucleon multiplicities for each number of wounded target nucleons.

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The superscripts ($\pm$) describe interactions which preserve and change the projectile isospin, respectively; the parameters β_ν specify the fraction of isospin preserve reactions. The $S^{\pm}_{\nu-1}(y)$ defines the probability of transition of a nucleon with longitudinal momentum fraction x/y to a state with longitudinal momentum x, after $(\nu-1)$ collisions. For the probability functions $S^{\pm}_{\nu}(y)$, we have for the first collision

$$S^{\pm}_0(y) = \frac{M_1^{\mathrm{p,n}}(y)}{\int_0^1 \mathrm{d}y M_1^{\mathrm{p,n}}(y)}, \tag{4}$$

and appropriate definitions of M_0^{p} and M_0^{n} [12].

Because it is assumed in this model that we may have different inelasticities upon subsequent collisions, a power law form is adopted for that difference, with an adjustable factor α_ν for $\nu > 1$,

$$S^{\pm}_\nu(y) = \frac{y^{\alpha_\nu} M_1^{\mathrm{p,n}}(y)}{\int_0^1 \mathrm{d}y y^{\alpha_\nu} M_1^{\mathrm{p,n}}(y)}. \tag{5}$$

If $\alpha_{\nu-1} = 0$, the distribution and the inelasticity for the νth collision is the same as for the first one.

Let us now consider the analytical solution for the nucleonic diffusion equation in the atmosphere, described by the expression

$$F_{\mathrm{N}}(E,t) = N_0 E^{-(\gamma+1)} \exp\left[-\frac{t}{\Lambda}\right] \tag{6}$$

where N_0 is the coefficient of the primary spectrum [3–5] and Λ is the attenuation length given by [3–5]

$$\frac{1}{\Lambda} = \frac{\sigma_{\mathrm{in}}^{\mathrm{N-ar}}(1 - \langle x^\gamma \rangle_{\mathrm{N-air}})}{24100} \quad (\mathrm{g/cm^2})^{-1}. \tag{7}$$

$\sigma_{\mathrm{in}}^{\mathrm{N-ar}}$ is the inelastic cross section and $\langle x^\gamma \rangle_{\mathrm{N-air}}$ is the γth moment of the nucleon–air leading-particle distribution.

In order to compute the moment $\langle x^\gamma \rangle_{\mathrm{N-air}}$, we use the Glauber model. The nucleon–air leading particle can be obtained by means of

$$M_{\mathrm{N-air}} = \sum P_\nu M_\nu, \tag{8}$$

where P_ν is the probability of ν-fold collisions of the primary nucleon inside the nucleus, and is given by

$$P_\nu = \frac{\int \mathrm{d}^2 b P_\nu(b)}{\sigma_{\mathrm{in}}^{\mathrm{N-air}}} \tag{9}$$

and

$$P_\nu(b) = \frac{1}{\nu!}\left[\sigma_{\mathrm{tot}}^{\mathrm{pp}} T(b)\right]^\nu \exp[-\sigma_{\mathrm{tot}}^{\mathrm{pp}} T(b)] \tag{10}$$

where $T(b)$ is the nuclear thickness.

From (1) and (2) we have, for the first collision,

$$\langle x^\gamma \rangle_1^{\mathrm{N}} = n_1^{\mathrm{p}} \langle x^\gamma \rangle_1^{\mathrm{p}} + n_1^{\mathrm{n}} \langle x^\gamma \rangle_1^{\mathrm{n}} \tag{11}$$

and, for $\nu > 1$,

$$\begin{aligned}\langle x^\gamma \rangle_\nu^{\mathrm{N}} &= n_\nu^{\mathrm{p}} \langle x^\gamma \rangle_\nu^{\mathrm{p}} + n_\nu^{\mathrm{n}} \langle x^\gamma \rangle_\nu^{\mathrm{n}} \\ &= \left[n_{\nu-1}^{\mathrm{p}} \langle x^\gamma \rangle_{\nu-1}^{\mathrm{p}} + n_{\nu-1}^{\mathrm{n}} \langle x^\gamma \rangle_{\nu-1}^{\mathrm{n}}\right] K_{\nu-1}\end{aligned} \tag{12}$$

with

$$\begin{aligned}K_{\nu-1}(\gamma) = \beta_{\nu-1} &\int_0^1 \mathrm{d}y y^\gamma S_\nu^+(y) + (1-\beta_{\nu-1}) \\ &\times \int_0^1 \mathrm{d}y y^\gamma S_\nu^-(y)\end{aligned} \tag{13}$$

being the elasticity for the γth moment of the νth collision. For $\gamma = 1$, (11) gives the usual nucleonic elasticity.

We shall assume [12] that the $S_\nu^{\pm}(y)$ are the same for all interactions with more than one collision, $\nu > 1$. In that situation K_ν is independent of ν,

$$K_\nu(\gamma) \to K(\gamma), \tag{14}$$

and, making use of a recurrence relation, we get

$$\langle x^\gamma \rangle_\nu^{\mathrm{N}} = \langle x^\gamma \rangle_1^{\mathrm{N}} K^{\nu-1} \tag{15}$$

On the other hand,

$$\langle x^\gamma \rangle_{\mathrm{p}}^{\mathrm{N}} = \langle x^\gamma \rangle_1^{\mathrm{N}} \sum_{\nu=1} P_\nu K^\nu. \tag{16}$$

Defining now

$$\eta = \frac{\langle x^\gamma \rangle_1^{\mathrm{N}}}{K}, \tag{17}$$

we obtain, from (8) and (15),

$$\begin{aligned}&\sigma_{\mathrm{in}}^{\mathrm{N-air}}(1 - \langle x^\gamma \rangle_{\mathrm{N-air}}) \\ &= \int \mathrm{d}^2 b[1 - \{\eta(\gamma)\exp[-(1-K(\gamma))\sigma_{\mathrm{tot}}^{\mathrm{pp}} T(b)] \\ &\quad +(1-\eta(\gamma))\exp[-\sigma_{\mathrm{tot}}^{\mathrm{pp}} T(b)]\}]\end{aligned} \tag{18}$$

This parameter η defines the relationship between the ν elasticity in the first interaction to the one for the successive interactions of protons and neutrons with the nucleus of the atmosphere. We note that for $\gamma = 1$, (18) allows us to calculate the average nucleon–air inelasticity:

$$\begin{aligned}&\sigma_{\mathrm{in}}^{\mathrm{N-air}}(1 - \langle x \rangle_{\mathrm{N-air}}) \\ &= \int \mathrm{d}^2 b[1 - \{\eta(1)\exp[-(1-K(1))\sigma_{\mathrm{tot}}^{\mathrm{pp}} T(b)] \\ &\quad +(1-\eta(1))\exp[-\sigma_{\mathrm{tot}}^{\mathrm{pp}} T(b)]\}]\end{aligned} \tag{19}$$

We tested (18) in comparison with cosmic ray data on nucleonic flux and hadronic flux in the atmosphere [13–16]. For $\sigma_{\mathrm{tot}}^{\mathrm{pp}}$, we used the UA4/2 Collaboration parametrization [17] and estimated $\sigma_{\mathrm{in}}^{\mathrm{N-air}}$ by means of the Glauber model [18], and for the $T(b)$ nuclear thickness we used the Woods–Saxon model [19]. The parameters η and K were left free. The best fit ($\aleph^2$/d.o.f. = 2.61) corresponds to $\eta = 1$ and $K = 0.34$ and is shown in Fig. 1 (nucleonic flux at sea level) [13,14], Fig. 2 (hadronic flux at sea level) [15], and Fig. 3 (hadronic flux at $t = 840$ g/cm^2) [16]. The hadronic flux was obtained by multiplication of the nucleonic flux in (6) by the Kascade factor [15]

$$R = \frac{\pi^+ + \pi^-}{\mathrm{p} + \mathrm{n}} = 0.04 + 0.27\ln(E/\mathrm{GeV}) \tag{20}$$

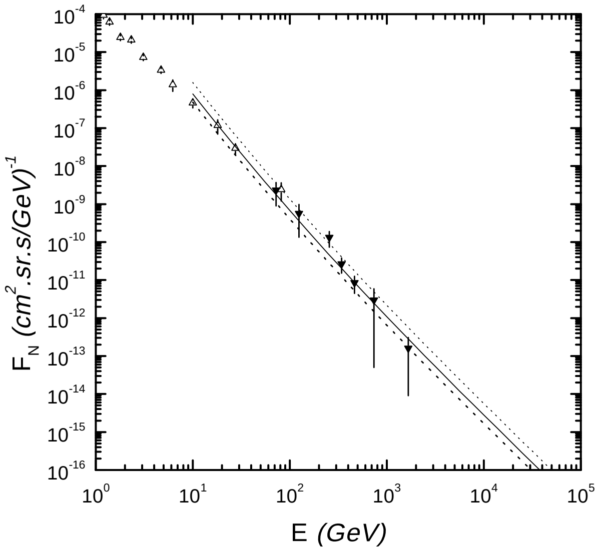

Fig. 1. Nucleonic flux at sea level. Experimental data from [13,14]. Continuous line: result of fit. Dashed lines: maximal and minimal values of the calculated nucleonic flux

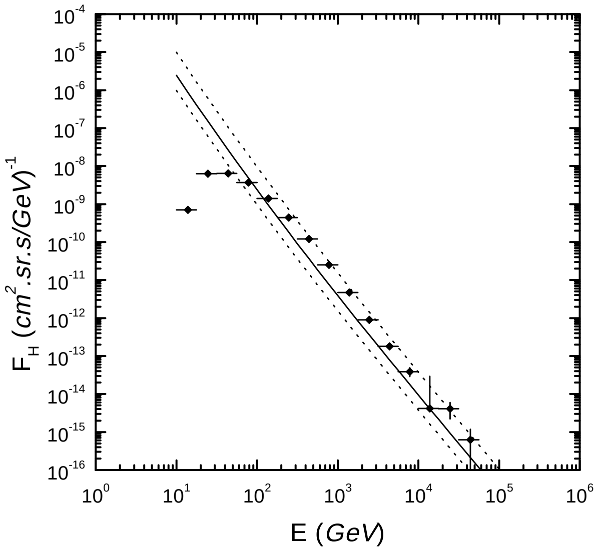

Fig. 3. Hadronic flux at $t = 840$ g/cm^2. Experimental data from [16]. Solid line: result of fit. Dashed lines: maximal and minimal values of the calculated hadronic flux

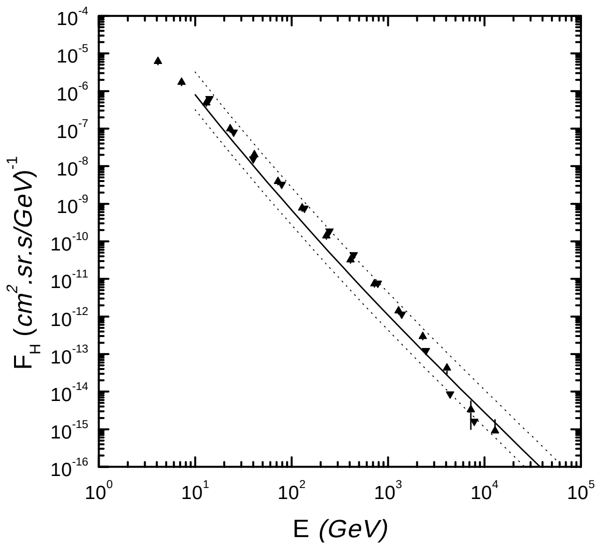

Fig. 2. Hadronic flux at sea level. Experimental data from [15]. Solid line: result of fit. Dashed lines: maximal and minimal values of the calculated hadronic flux

to count the number of pions in the hadronic flux. We have used the same primary spectrum as in [19]. We also show in these figures the maximal and minimal values for the flux considering the experimental errors in the primary spectrum.

3 Conclusions

We have here presented an analysis of the hadronic flux in the atmosphere in the Glauber framework model, using an iterative leading-particle model to relate the moment of the leading-particle distribution in nucleon–air collisions with that of nucleon–proton interactions. In the analysis of the data, we did not use any parametrization for $M_\nu^{\mathrm{N}}(x)$. We have simply used (5) and (18) in comparison with data to extract η and K. The fact that η is equal to 1 means that after the first collision, the elasticity remains the same (it does not increase as found in [12]). The value of K, which is smaller than 0.5, is expected in view of the inclusion of the leading neutrons. We note that the obtained value of K, the average elasticity in nucleon–proton collisions, is in agreement with the value obtained in [12] and by Jones [20] in the analysis of inclusive $\mathrm{p+p} \to \mathrm{p}+X$ reactions. However, the average N–air elasticity will not be constant with energy, because of (19). A careful discussion of this problem was recently done by Bellandi, et al. [21]

One point should be stressed. In [12], the p_t dependence of the leading-particle distribution was included in an *ad hoc* manner, and assumed to be the same for $\nu > 1$. This assumption does not seem consistent with the multicollision Glauber framework used by these authors. It does not seem physically possible to have multiple scattering with energy loss in the forward direction but without transverse momentum spread.

In order to make the point clear, let us write the ν-collision inclusive leading-particle distribution, for $\nu > 1$, in the form

$$\frac{1}{\sigma_\nu^{\mathrm{p-A}}} \frac{\mathrm{d}^3\sigma^{\mathrm{pA}\to\mathrm{N}}}{\mathrm{d}p_t^2\mathrm{d}x} = M_\nu^{\mathrm{N}}(x) \frac{\langle b_\nu^{\mathrm{N}} \rangle}{\pi} \exp[-\langle b_\nu^{\mathrm{N}} \rangle p_t^2]. \qquad (21)$$

In [12] the $M_\nu^{\mathrm{N}}(x)$ decrease not very fast with x (because elasticity increases for $\nu > 1$) and, for the fixed p_t, x distribution, this is compensated by some average slope parameter $\langle b_\nu^{\mathrm{N}} \rangle$. In our case, $M_\nu^{\mathrm{N}}(x)$ decreases fast with x (elasticity is always the same) but for small values of ν (the dominant contributions), this is compensated by a larger value of b_ν^{N} (b_ν^{N} in the Glauber framework decrease with ν). It is clear that we can as well fit fixed p_t, x distributions. We did not attempt to do that, because the correct procedure should be to use the convolution in 3 dimensions to analyze $\mathrm{p} + \mathrm{A} \to \mathrm{N}(\text{nucleon}) + X$ inclusive reactions.

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